Mathematics Department

Twyford CE High School

Twyford Crescent, London, W3 9PP www.twyford.ealing.sch.uk

An introduction to

A Level Mathematics

This induction booklet is for students intending to begin studying A Level Maths in Year 12 from next September.

It is important that you are able to work at this standard – read the introduction carefully, and spend time working through the exercises before you start in September.

Introduction to A Level Maths at Twyford

Thank you for choosing to study Mathematics in the Sixth Form at Twyford CE High School.

If you are studying Mathematics on its own, you will sit three internal exams at the end of year 12, which will assess a combination of three areas: Pure maths, Statistics and Mechanics. At the end of year 13 you will sit 3 external exams that will assess content covered during both years with the same combination of Pure maths, Statistics and Mechanics. If you have chosen to study Further Mathematics as well, you will sit three internal exams at the end of year 12, which will assess the content of a full Maths A level, assessing a combination of Pure maths, Statistics and Mechanics. During year 13 you will cover the content of the Further Maths A level which is a combination of Pure Maths, Decision maths and Mechanics. At the end of year 13 you will sit 6 external exams, 3 which will assess the content covered in the Maths A level (what you covered in year 12) and 3 which will assess the content covered in the Further Maths A level (what you covered in year 13).

The Mathematics Department is committed to ensuring that you make good progress throughout your A level course. In order that you make the best possible start to the course, we have placed this booklet on our website. It is important that you spend time working through the topics in this booklet over the summer, as you need to have a good knowledge of these topics <u>before</u> you commence your course in September. You should have met all the topics before at GCSE. The answers to the exercises are at the back of the booklet. You will need to be much organised, so keep your work in a folder and mark any queries to ask at the beginning of term. This booklet is largely concerned with algebraic manipulation, but you also need to be able to calculate with fractions – which you may not have used for a while – as fraction calculations are taken for granted in many aspects of A Level Mathematics.

In the second week of term, you will take a test in class to check how well you understand these topics, so it is important that you thoroughly understand the content of the booklet by then. If you do not pass this test, this will be considered a serious issue, and it will beg the question as to whether you should be taking this subject. (Students have been taken off the course in the past during Year 12 and have therefore not gained A Level Mathematics). In particular, please note that there is a mock test provided at the back of this booklet, which is similar in style to the test that you will be given in class, and which you must submit in your first lesson.

Use this introduction to give you a good start to your A level work so that you enjoy, and benefit from, the course. The more effort you put in, right from the start, the better you should do.

Mr M Harley	**Ms L Ara**
Head of Mathematics	**Head of KS5 Maths**

<u>Please note</u>:

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Section 1: REMOVING BRACKETS

To remove a single bracket multiply every term in the bracket by the number or expression outside:

Examples

1) $3(x + 2y) = 3x + 6y$

2) $-2(2x - 3) = (-2)(2x) + (-2)(-3)$
$= -4x + 6$

To expand two brackets multiply everything in the first bracket by everything in the second bracket. You may have used
* the smiley face method
* FOIL (First Outside Inside Last)
* using a grid.

Examples:

1) $(x + 1)(x + 2) = x(x + 2) + 1(x + 2)$

or

$(x + 1)(x + 2) = x^2 + 2 + 2x + x$
$= x^2 + 3x + 2$

or

	x	1
x	x^2	x
2	2x	2

$(x + 1)(x + 2) = x^2 + 2x + x + 2$
$= x^2 + 3x + 2$

2) $(x - 2)(2x + 3) = x(2x + 3) - 2(2x + 3)$
$= 2x^2 + 3x - 4x - 6$
$= 2x^2 - x - 6$

or

$(x - 2)(2x + 3) = 2x^2 - 6 + 3x - 4x = 2x^2 - x - 6$

or

	x	-2
2x	$2x^2$	-4x
3	3x	-6

$(2x + 3)(x - 2) = 2x^2 + 3x - 4x - 6$
$= 2x^2 - x - 6$

EXERCISE A Multiply out the following brackets and simplify.

1. $7(4x + 5)$
2. $-3(5x - 7)$
3. $5a - 4(3a - 1)$
4. $4y + y(2 + 3y)$
5. $-3x - (x + 4)$
6. $5(2x - 1) - (3x - 4)$
7. $(x + 2)(x + 3)$
8. $(t - 5)(t - 2)$
9. $(2x + 3y)(3x - 4y)$
10. $4(x - 2)(x + 3)$
11. $(2y - 1)(2y + 1)$
12. $(3 + 5x)(4 - x)$

Two Special Cases

Perfect Square:
$(x + a)^2 = (x + a)(x + a) = x^2 + 2ax + a^2$
$(2x - 3)^2 = (2x - 3)(2x - 3) = 4x^2 - 12x + 9$

Difference of two squares:
$(x - a)(x + a) = x^2 - a^2$
$(x - 3)(x + 3) = x^2 - 3^2$
$= x^2 - 9$

EXERCISE B Expand the following

1. $(x - 1)^2$
2. $(3x + 5)^2$
3. $(7x - 2)^2$
4. $(x + 2)(x - 2)$
5. $(3x + 1)(3x - 1)$
6. $(5y - 3)(5y + 3)$

Section 2: LINEAR EQUATIONS

When solving an equation whatever you do to one side must also be done to the other. You may
- add the same amount to both side
- subtract the same amount from each side
- multiply the whole of each side by the same amount
- divide the whole of each side by the same amount.

If the equation has unknowns on both sides, collect all the letters onto the same side of the equation.

If the equation contains brackets, you often start by expanding the brackets.

A linear equation contains only numbers and terms in x. (Not x^2 or x^3 or 1/x etc)

More help on solving equations can be obtained by downloading the leaflet available at this website:
http://www.mathcentre.ac.uk/resources/workbooks/mathcentre/web-simplelinear.pdf

Example 1: Solve the equation $64 – 3x = 25$

Solution: There are various ways to solve this equation. One approach is as follows:

Step 1: Add 3x to both sides (so that the x term is positive): $64 = 3x + 25$

Step 2: Subtract 25 from both sides: $39 = 3x$

Step 3: Divide both sides by 3: $13 = x$

So the solution is $x = 13$.

Example 2: Solve the equation $6x + 7 = 5 – 2x$.

Solution:
Step 1: Begin by adding 2x to both sides $8x + 7 = 5$
(to ensure that the x terms are together on the same side)

Step 2: Subtract 7 from each side: $8x = -2$

Step 3: Divide each side by 8: $x = -\frac{1}{4}$

Exercise A: Solve the following equations, showing each step in your working:

1) $2x + 5 = 19$
2) $5x – 2 = 13$
3) $11 – 4x = 5$
4) $5 – 7x = -9$
5) $11 + 3x = 8 – 2x$
6) $7x + 2 = 4x – 5$

Example 3: Solve the equation $2(3x – 2) = 20 – 3(x + 2)$

Step 1: Multiply out the brackets: $6x – 4 = 20 – 3x – 6$
(taking care of the negative signs)

Step 2: Simplify the right hand side: $6x – 4 = 14 – 3x$

Step 3: Add 3x to each side: $9x – 4 = 14$

Step 4: Add 4: $9x = 18$

Step 5: Divide by 9: $x = 2$

Exercise B: Solve the following equations.

1) $5(2x – 4) = 4$
2) $4(2 – x) = 3(x – 9)$
3) $8 – (x + 3) = 4$
4) $14 – 3(2x + 3) = 2$

EQUATIONS CONTAINING FRACTIONS

When an equation contains a fraction, the first step is usually to multiply through by the denominator of the fraction. This ensures that there are no fractions in the equation.

Example 4: Solve the equation $\frac{y}{2}+5=11$

Solution:

Step 1: Multiply through by 2 (the denominator in the fraction): $y+10=22$

Step 2: Subtract 10: y = 12

Example 5: Solve the equation $\frac{1}{3}(2x+1)=5$

Solution:

Step 1: Multiply by 3 (to remove the fraction) $2x+1=15$

Step 2: Subtract 1 from each side 2x = 14

Step 3: Divide by 2 x = 7

When an equation contains two fractions, you need to multiply by the lowest common denominator. This will then remove both fractions.

Example 6: Solve the equation $\frac{x+1}{4}+\frac{x+2}{5}=2$

Solution:

Step 1: Find the lowest common denominator: The smallest number that both 4 and 5 divide into is 20.

Step 2: Multiply both sides by the lowest common denominator $\frac{20(x+1)}{4}+\frac{20(x+2)}{5}=40$

Step 3: Simplify the left hand side: $\frac{\overset{5}{\cancel{20}}(x+1)}{\cancel{4}}+\frac{\overset{4}{\cancel{20}}(x+2)}{\cancel{5}}=40$

5(x + 1) + 4(x + 2) = 40

Step 4: Multiply out the brackets: 5x + 5 + 4x + 8 = 40

Step 5: Simplify the equation: 9x + 13 = 40

Step 6: Subtract 13 9x = 27

Step 7: Divide by 9: x = 3

Example 7: Solve the equation $x+\frac{x-2}{4}=2-\frac{3-5x}{6}$

Solution: The lowest number that 4 and 6 go into is 12. So we multiply every term by 12:

$$12x+\frac{12(x-2)}{4}=24-\frac{12(3-5x)}{6}$$

Simplify $12x+3(x-2)=24-2(3-5x)$

Expand brackets $12x+3x-6=24-6+10x$

Simplify $15x-6=18+10x$

Subtract 10x $5x-6=18$

Add 6 5x = 24

Divide by 5 x = 4.8

Exercise C: Solve these equations

1) $\frac{1}{2}(x+3)=5$

2) $\frac{2x}{3}-1=\frac{x}{3}+4$

3) $\frac{y}{4}+3=5-\frac{y}{3}$

4) $\frac{x-2}{7}=2+\frac{3-x}{14}$

5) $\frac{7x-1}{2}=13-x$

6) $\frac{y-1}{2}+\frac{y+1}{3}=\frac{2y+5}{6}$

7) $2x+\frac{x-1}{2}=\frac{5x+3}{3}$

8) $2-\frac{5}{x}=\frac{10}{x}-1$

FORMING EQUATIONS

Example 8: Find three consecutive numbers so that their sum is 96.

Solution: Let the first number be n, then the second is n + 1 and the third is n + 2.
Therefore n + (n + 1) + (n + 2) = 96
3n + 3 = 96
3n = 93
n = 31
So the numbers are 31, 32 and 33.

Exercise D:

1) Find 3 consecutive even numbers so that their sum is 108.

2) The perimeter of a rectangle is 79 cm. One side is three times the length of the other. Form an equation and hence find the length of each side.

3) Two girls have 72 photographs of celebrities between them. One gives 11 to the other and finds that she now has half the number her friend has.
Form an equation, letting n be the number of photographs one girl had at the **beginning**.
Hence find how many each has **now**.

Section 3: SIMULTANEOUS EQUATIONS

Example

$$3x + 2y = 8 \quad ①$$
$$5x + y = 11 \quad ②$$

x and y stand for two numbers. Solve these equations in order to find the values of x and y by eliminating one of the letters from the equations.

In these equations it is simplest to eliminate y. Make the coefficients of y the same in both equations. To do this multiply equation ② by 2, so that both equations contain 2y:

$$3x + 2y = 8 \quad ①$$
$$10x + 2y = 22 \quad 2×② = ③$$

To eliminate the y terms, subtract equation ③ from equation ①. We get: $7x = 14$

i.e. $x = 2$

To find y substitute x = 2 into one of the original equations. For example put it into ②:

$$10 + y = 11$$
$$y = 1$$

Therefore the solution is $x = 2$, $y = 1$.

Remember: Check your solutions by substituting both x and y into the original equations.

Example: Solve

$$2x + 5y = 16 \quad ①$$
$$3x – 4y = 1 \quad ②$$

Solution: Begin by getting the same number of x or y appearing in both equation. For example, multiply the top equation by 4 and the bottom equation by 5 to get 20y in both equations:

$$8x + 20y = 64 \quad ③$$
$$15x – 20y = 5 \quad ④$$

As the SIGNS in front of 20y are DIFFERENT, eliminate the y terms from the equations by ADDING:

$$23x = 69 \quad ③+④$$

i.e. $x = 3$

Substituting this into equation ① gives:

$$6 + 5y = 16$$
$$5y = 10$$

So… $y = 2$

The solution is $x = 3$, $y = 2$.

If you need **more help** on solving simultaneous equations, you can download a booklet from the following website:

http://www.mathcentre.ac.uk/resources/workbooks/mathcentre/web-simultaneous1.pdf

Exercise: Solve the pairs of simultaneous equations in the following questions:

1) $x + 2y = 7$
$3x + 2y = 9$

2) $x + 3y = 0$
$3x + 2y = -7$

3) $3x - 2y = 4$
$2x + 3y = -6$

4) $9x - 2y = 25$
$4x - 5y = 7$

5) $4a + 3b = 22$
$5a - 4b = 43$

6) $3p + 3q = 15$
$2p + 5q = 14$

Section 4: FACTORISING

Taking out a common factor

Example 1: Factorise $12x - 30$

Solution: 6 is a common factor to both 12 and 30. Factorise by taking 6 outside a bracket:

$$12x - 30 = 6(2x - 5)$$

Example 2: Factorise $6x^2 - 2xy$

Solution: 2 is a common factor to both 6 and 2. Both terms also contain an x.
Factorise by taking 2x outside a bracket.

$$6x^2 - 2xy = 2x(3x - y)$$

Example 3: Factorise $9x^3y^2 - 18x^2y$

Solution: 9 is a common factor to both 9 and 18.
The highest power of x that is present in both expressions is x^2.
There is also a y present in both parts.
So we factorise by taking $9x^2y$ outside a bracket:

$$9x^3y^2 - 18x^2y = 9x^2y(xy - 2)$$

Example 4: Factorise $3x(2x - 1) - 4(2x - 1)$

Solution: There is a common bracket as a factor.
So we factorise by taking $(2x - 1)$ out as a factor.
The expression factorises to $(2x - 1)(3x - 4)$

Exercise A Factorise each of the following

1) $3x + xy$

3) $pq^2 - p^2q$

2) $4x^2 - 2xy$

4) $3pq - 9q^2$

5) $2x^3 - 6x^2$

6) $8a^5b^2 - 12a^3b^4$

7) $5y(y - 1) + 3(y - 1)$

Factorising quadratics

Simple quadratics: Factorising quadratics of the form $x^2 + bx + c$
The method is:
Step 1: Form two brackets (x …)(x …)
Step 2: Find two numbers that multiply to give c and add to make b. Write these two numbers at the end of the brackets.

Example 1: Factorise $x^2 - 9x - 10$.

Solution: Find two numbers that multiply to make -10 and add to make -9. These numbers are -10 and 1.
Therefore $x^2 - 9x - 10 = (x - 10)(x + 1)$.

General quadratics: Factorising quadratics of the form $ax^2 + bx + c$
One method (you may not have seen) is:
Step 1: Find two numbers that multiply together to make ac and add to make b.
Step 2: Split up the bx term using the numbers found in step 1.
Step 3: Factorise the front and back pair of expressions as fully as possible.
Step 4: There should be a common bracket. Take this out as a common factor.

Example 2: Factorise $6x^2 + x - 12$.

Solution: We need to find two numbers that multiply to make $6 \times -12 = -72$ and add to make 1. These two numbers are -8 and 9.

Therefore, $6x^2 + x - 12 = \underbrace{6x^2 - 8x}\ \underbrace{+ 9x - 12}$

$= 2x(3x - 4) + 3(3x - 4)$ (the two brackets must be identical)

$= (3x - 4)(2x + 3)$

Difference of two squares: Factorising quadratics of the form $x^2 - a^2$

Remember that $x^2 - a^2 = (x + a)(x - a)$.

Therefore: $x^2 - 9 = x^2 - 3^2 = (x+3)(x-3)$
$16x^2 - 25 = (2x)^2 - 5^2 = (2x+5)(2x-5)$

Also notice that: $2x^2 - 8 = 2(x^2 - 4) = 2(x+4)(x-4)$
and $3x^3 - 48xy^2 = 3x(x^2 - 16y^2) = 3x(x+4y)(x-4y)$

Factorising by pairing or grouping
Factorise expressions like $2x^2 + xy - 2x - y$ using the method of factorising by pairing:

$2x^2 + xy - 2x - y = x(2x + y) - 1(2x + y)$ (factorise front and back pairs, both brackets identical)
$= (2x + y)(x - 1)$

If you need **more help** with factorising, you can download a booklet from this website:
http://www.mathcentre.ac.uk/resources/workbooks/mathcentre/web-factorisingquadratics.pdf

Exercise B Factorise

1) $x^2 - x - 6$

2) $x^2 + 6x - 16$

3) $2x^2 + 5x + 2$

4) $2x^2 - 3x$

5) $3x^2 + 5x - 2$

6) $2y^2 + 17y + 21$

7) $7y^2 - 10y + 3$

8) $10x^2 + 5x - 30$

9) $4x^2 - 25$

10) $x^2 - 3x - xy + 3y^2$

11) $4x^2 - 12x + 8$

12) $16m^2 - 81n^2$

13) $4y^3 - 9a^2y$

14) $8(x+1)^2 - 2(x+1) - 10$

Section 5: CHANGING THE SUBJECT OF A FORMULA

Rearranging a formula is similar to solving an equation –always do the same to both sides.

Example 1: Make x the subject of the formula y = 4x + 3.

Solution: y = 4x + 3

Subtract 3 from both sides: y – 3 = 4x

Divide both sides by 4; $\frac{y-3}{4} = x$

So $x = \frac{y-3}{4}$ is the same equation but with x the subject.

Example 2: Make x the subject of y = 2 – 5x

Solution: Notice that in this formula the x term is negative.

y = 2 – 5x

Add 5x to both sides y + 5x = 2 (the x term is now positive)

Subtract y from both sides 5x = 2 – y

Divide both sides by 5 $x = \frac{2-y}{5}$

Example 3: The formula $C = \frac{5(F-32)}{9}$ is used to convert between ° Fahrenheit and ° Celsius.

Rearrange to make F the subject.

$$C = \frac{5(F-32)}{9}$$

Multiply by 9 $9C = 5(F-32)$ (this removes the fraction)

Expand the brackets $9C = 5F - 160$

Add 160 to both sides $9C + 160 = 5F$

Divide both sides by 5 $\frac{9C+160}{5} = F$

Therefore the required rearrangement is $F = \frac{9C+160}{5}$

Exercise A Make x the subject of each of these formulae:

1) y = 7x – 1

2) $y = \frac{x+5}{4}$

3) $4y = \frac{x}{3} - 2$

4) $y = \frac{4(3x-5)}{9}$

Example 4: Make x the subject of $x^2 + y^2 = w^2$

Solution: $x^2 + y^2 = w^2$

Subtract y^2 from both sides: $x^2 = w^2 - y^2$ (this isolates the term involving x)

Square root both sides: $x = \pm\sqrt{w^2 - y^2}$

Remember the positive & negative square root.

Example 5: Make a the subject of the formula $t = \frac{1}{4}\sqrt{\frac{5a}{h}}$

Solution: $t = \frac{1}{4}\sqrt{\frac{5a}{h}}$

Multiply by 4 $4t = \sqrt{\frac{5a}{h}}$

Square both sides $16t^2 = \frac{5a}{h}$

Multiply by h: $16t^2 h = 5a$

Divide by 5: $\frac{16t^2 h}{5} = a$

Exercise B: Make t the subject of each of the following

1) $P = \frac{wt}{32r}$

2) $P = \frac{wt^2}{32r}$

3) $V = \frac{1}{3}\pi t^2 h$

4) $P = \sqrt{\frac{2t}{g}}$

5) $Pa = \frac{w(v-t)}{g}$

6) $r = a + bt^2$

Harder examples

Sometimes the subject occurs in more than one place in the formula. In these questions collect the terms involving this variable on one side of the equation, and put the other terms on the opposite side.

Example 6: Make t the subject of the formula $a - xt = b + yt$

Solution: $a - xt = b + yt$

Start by collecting all the t terms on the right hand side:

Add xt to both sides: $a = b + yt + xt$

Now put the terms without a t on the left hand side:

Subtract b from both sides: $a - b = yt + xt$

Factorise the RHS: $a - b = t(y + x)$

Divide by (y + x): $\frac{a-b}{y+x} = t$

So the required equation is $t = \frac{a-b}{y+x}$

Example 7: Make W the subject of the formula $T - W = \frac{Wa}{2b}$

Solution: This formula is complicated by the fractional term. Begin by removing the fraction:

Multiply by 2b: $2bT - 2bW = Wa$

Add 2bW to both sides: $2bT = Wa + 2bW$ (this collects the W's together)

Factorise the RHS: $2bT = W(a + 2b)$

Divide both sides by a + 2b: $W = \frac{2bT}{a+2b}$

If you need more help you can download an information booklet on rearranging equations from the following website: http://www.mathcentre.ac.uk/resources/workbooks/mathcentre/web-formulae2-tom.pdf

Exercise C Make x the subject of these formulae:

1) $ax + 3 = bx + c$

2) $3(x + a) = k(x - 2)$

3) $y = \frac{2x+3}{5x-2}$

4) $\frac{x}{a} = 1 + \frac{x}{b}$

Section 6: SOLVING QUADRATIC EQUATIONS

A quadratic equation has the form $ax^2 + bx + c = 0$.

There are two methods that are commonly used for solving quadratic equations:
* factorising
* the quadratic formula

Not all quadratic equations can be solved by factorising.

Method 1: Factorising

Make sure that the equation is rearranged so that the right hand side is 0. It usually makes it easier if the coefficient of x^2 is positive.

Example 1 : Solve $x^2 - 3x + 2 = 0$

Factorise $(x - 1)(x - 2) = 0$
Either $(x - 1) = 0$ or $(x - 2) = 0$
So the solutions are $x = 1$ or $x = 2$

Note: The individual values $x = 1$ and $x = 2$ are called the **roots** of the equation.

Example 2: Solve $x^2 - 2x = 0$

Factorise: $x(x - 2) = 0$
Either $x = 0$ or $(x - 2) = 0$
So $x = 0$ or $x = 2$

Method 2: Using the quadratic formula

The roots of the quadratic equation $ax^2 + bx + c = 0$ are given by the formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Example 3: Solve the equation $2x^2 - 5 = 7 - 3x$

Solution: First we rearrange so that the right hand side is 0. We get $2x^2 + 3x - 12 = 0$
We can then tell that a = 2, b = 3 and c = -12.
Substituting these into the quadratic formula gives:

$$x = \frac{-3 \pm \sqrt{3^2 - 4 \times 2 \times (-12)}}{2 \times 2} = \frac{-3 \pm \sqrt{105}}{4}$$ (this is the surd form for the solutions)

If we have a calculator, we can evaluate these roots to get: $x = 1.81$ or $x = -3.31$

If you need more help with the work in this chapter, there is an information booklet downloadable from this web site:
http://www.mathcentre.ac.uk/resources/workbooks/mathcentre/web-quadraticequations.pdf

EXERCISE

1) Use factorisation to solve the following equations: a) $x^2 + 3x + 2 = 0$

b) $x^2 - 3x - 4 = 0$ c) $x^2 = 15 - 2x$

2) Find the roots of the following equations: a) $x^2 + 3x = 0$

b) $x^2 - 4x = 0$ c) $4 - x^2 = 0$

3) Solve the following equations either by factorising or by using the formula:

a) $6x^2 - 5x - 4 = 0$ b) $8x^2 - 24x + 10 = 0$

4) Use the formula to solve the following equations to 3 significant figures where possible

a) $x^2 + 7x + 9 = 0$ b) $6 + 3x = 8x^2$

c) $4x^2 - x - 7 = 0$ d) $x^2 - 3x + 18 = 0$

e) $3x^2 + 4x + 4 = 0$ f) $3x^2 = 13x - 16$

Section 7: INDICES

Basic rules of indices

y^4 means $y \times y \times y \times y$. 4 is called the **index** (plural: indices) or **exponent** of y.

There are 3 basic rules of indices:

1) $a^m \times a^n = a^{m+n}$ e.g. $3^4 \times 3^5 = 3^9$
2) $a^m \div a^n = a^{m-n}$ e.g. $3^8 \div 3^3 = 3^5$
3) $(a^m)^n = a^{mn}$ e.g. $\left(3^2\right)^5 = 3^{10}$

Further examples

$y^4 \times 5y^3 = 5y^7$

$4a^3 \times 6a^2 = 24a^5$ (multiply the numbers and multiply the a's)

$2c^2 \times \left(-3c^6\right) = -6c^8$ (multiply the numbers and multiply the c's)

$24d^7 \div 3d^2 = \dfrac{24d^7}{3d^2} = 8d^5$ (divide the numbers and divide the d terms by subtracting the powers)

Exercise A Simplify the following: Remember that $b = b^1$

1) $b \times 5b^5$
2) $3c^2 \times 2c^5$
3) $b^2c \times bc^3$
4) $2n^6 \times (-6n^2)$
5) $8n^8 \div 2n^3$
6) $d^{11} \div d^9$
7) $\left(a^3\right)^2$
8) $\left(-d^4\right)^3$

Zero index: Remember $a^0 = 1$ **F**or any non-zero number, a.

Therefore $5^0 = 1$ $\left(\frac{3}{4}\right)^0 = 1$ $(-5.2304)^0 = 1$

Negative powers

A power of -1 corresponds to the reciprocal of a number, i.e. $a^{-1} = \frac{1}{a}$

Therefore $5^{-1} = \frac{1}{5}$

$0.25^{-1} = \frac{1}{0.25} = 4$

$\left(\frac{4}{5}\right)^{-1} = \frac{5}{4}$ (Find the reciprocal of a fraction by turning it upside down)

This result can be extended to more general negative powers: $a^{-n} = \frac{1}{a^n}$.

This means:

$3^{-2} = \frac{1}{3^2} = \frac{1}{9}$ $2^{-4} = \frac{1}{2^4} = \frac{1}{16}$ $\left(\frac{1}{4}\right)^{-2} = \left(\left(\frac{1}{4}\right)^{-1}\right)^2 = \left(\frac{4}{1}\right)^2 = 16$

Fractional powers:

Fractional powers correspond to roots: $a^{1/2} = \sqrt{a}$ $a^{1/3} = \sqrt[3]{a}$ $a^{1/4} = \sqrt[4]{a}$

In general: $a^{1/n} = \sqrt[n]{a}$

Therefore:

$8^{1/3} = \sqrt[3]{8} = 2$ $25^{1/2} = \sqrt{25} = 5$ $10000^{1/4} = \sqrt[4]{10000} = 10$

A more general fractional power can be dealt with in the following way: $a^{m/n} = \left(a^{1/n}\right)^m$

So $4^{3/2} = \left(\sqrt{4}\right)^3 = 2^3 = 8$

$\left(\frac{8}{27}\right)^{2/3} = \left(\left(\frac{8}{27}\right)^{1/3}\right)^2 = \left(\frac{2}{3}\right)^2 = \frac{4}{9}$

$\left(\frac{25}{36}\right)^{-3/2} = \left(\frac{36}{25}\right)^{3/2} = \left(\sqrt{\frac{36}{25}}\right)^3 = \left(\frac{6}{5}\right)^3 = \frac{216}{125}$

Exercise B: Find the value of:

1) $4^{1/2}$

2) $27^{1/3}$

3) $\left(1/9\right)^{1/2}$

4) 5^{-2}

5) 18^0

6) 7^{-1}

7) $27^{2/3}$

8) $\left(\frac{2}{3}\right)^{-2}$

9) $8^{-2/3}$

10) $(0.04)^{1/2}$

11) $\left(\frac{8}{27}\right)^{2/3}$

12) $\left(\frac{1}{16}\right)^{-3/2}$

Simplify each of the following:

13) $2a^{1/2} \times 3a^{5/2}$

14) $x^3 \times x^{-2}$

15) $\left(x^2 y^4\right)^{1/2}$

Section 8: SURDS

A number is said to be in surd form if it involves as irrational root. Surds are used to gain exact answers rather than results that are given to a particular degree of accuracy such as 3 significant figures.

Key rules are that $\sqrt{ab} = \sqrt{a}\,\sqrt{b}$ and that $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

(NOTE: $\sqrt{a+b} \neq \sqrt{a} + \sqrt{b}$)

Example 1 : $\sqrt{18} = \sqrt{9}\,\sqrt{2} = 3\sqrt{2}$

Example 2: $\sqrt{0.16} = \sqrt{\frac{16}{100}} = \frac{\sqrt{16}}{\sqrt{100}} = \frac{4}{10} = 0.4$

Example 3: Simplify $(\sqrt{2} - 3)(\sqrt{2} + 5)$

$$\begin{aligned} &(\sqrt{2} - 3)(\sqrt{2} + 5) \\ &= 2 - 3\sqrt{2} + 5\sqrt{2} - 15 \\ &= 2\sqrt{2} - 13 \end{aligned}$$

Exercise A:

Write each of the following as an integer, a decimal or in the simplest surd form.

1) $\sqrt{81}$

2) $\sqrt{50 - 14}$

3) $3 - \sqrt{0.25}$

4) $5\sqrt{2.25}$

5) $(\sqrt{3})^3$

6) $\sqrt{50}$

7) $10\sqrt{0.49}$

8) $3\sqrt{54}$

9) $\frac{\sqrt{108}}{\sqrt{3}}$

10) $\sqrt{10}\,(\sqrt{5} + \sqrt{2})$

11) $(2 + 2\sqrt{2})(5 + \sqrt{2})$

Rationalising the denominator

This is a form of simplifying where you ensure that there are no irrational terms on the denominator of a fraction. In simple cases this is achieving by multiplying by the square root that you already have on the denominator.

Example 4: $\frac{2}{3\sqrt{5}} = \frac{2\sqrt{5}}{3\sqrt{5}\sqrt{5}} = \frac{2\sqrt{5}}{15}$

Example 5: Write $\sqrt{20} + \frac{30}{\sqrt{5}}$ in the form $n\sqrt{5}$, where n is an integer.

$\sqrt{20} = \sqrt{4}\sqrt{5} = 2\sqrt{5}$

$\frac{30}{\sqrt{5}} = \frac{30\sqrt{5}}{5} = 6\sqrt{5}$

So $\sqrt{20} + \frac{30}{\sqrt{5}} = 8\sqrt{5}$

Example 6: Rationalise the denominator of $\frac{4}{3-\sqrt{5}}$

$$\frac{4}{3-\sqrt{5}} = \frac{4}{3-\sqrt{5}} \times \frac{3+\sqrt{5}}{3+\sqrt{5}} = \frac{4(3+\sqrt{5})}{(3-\sqrt{5})(3+\sqrt{5})} = \frac{12+4\sqrt{5}}{9-5} = \frac{12+4\sqrt{5}}{4} = 3+\sqrt{5}$$

Example 7: Rationalise the denominator of $\frac{4+\sqrt{7}}{4+\sqrt{5}}$

$$\frac{4+\sqrt{7}}{4+\sqrt{5}} = \frac{4+\sqrt{7}}{4+\sqrt{5}} \times \frac{4-\sqrt{5}}{4-\sqrt{5}} = \frac{(4+\sqrt{7})(4-\sqrt{5})}{(4+\sqrt{5})(4-\sqrt{5})} = \frac{16-4\sqrt{5}+4\sqrt{7}-\sqrt{35}}{11}$$

Exercise B:

Rationalise the denominators of the following questions and simplify where necessary.

1) $\frac{4}{\sqrt{3}}$
2) $\frac{1}{\sqrt{10}}$
3) $\frac{8\sqrt{3}}{\sqrt{7}}$
4) $\frac{15-\sqrt{5}}{\sqrt{5}}$

5) $5\sqrt{3} + \frac{6}{\sqrt{3}}$
6) $\frac{20+\sqrt{10}}{\sqrt{5}}$
7) $4\sqrt{5}\,(3 + \frac{2}{\sqrt{10}})$
8) $\frac{5}{5+\sqrt{7}}$

9) $\frac{2}{4-\sqrt{2}}$
10) $\frac{3+\sqrt{7}}{2-\sqrt{2}}$
11) $\frac{5+\sqrt{5}}{3+\sqrt{5}}$

Section 9: PROOF

Using algebra to prove a statement is true.

Example 1: Prove that the product of any two odd numbers is odd.

If m and n are integers then 2n+1 and 2m+1 are odd numbers.
The product is $(2n+1)(2m+1)=4mn+2m+2n+1$
$=2(2mn+m+n)+1$, which is odd
So the product of the two odd numbers is odd.

Example 2: Prove that the sum of any five consecutive numbers is a multiple of 5.

Five consecutive numbers are n, n+1, n+2, n+3 and n+4 where n is an integer.
The sum is $n+(n+1)+(n+2)+(n+3)+(n+4)=5n+10$
$=5(n+2)$, which is a multiple of 5
So the sum of any five consecutive numbers is a multiple of 5.

Exercise A:

1) Prove that the sum of any two consecutive even numbers cannot be a multiple of 4.
2) Prove that the sum of any two consecutive odd numbers is a multiple of 4.
3) Prove that, for any for consecutive numbers, the product of the middle two numbers is always 2 more than the product of the first and last numbers.
4) Prove that the difference between two consecutive square numbers is always odd.

Maths Practice Test – answers to be submitted in the first lesson

(If you are taking Further Maths see the next page instead)

Your test in week 2 will ask similar questions to this test.

You may NOT use a calculator

1. Expand and simplify
   (a) $(2x + 3)(2x – 1)$ (b) $(a + 3)^2$ (c) $4x(3x – 2) – x(2x + 5)$

2. Factorise
   (a) $x^2 – 7x$ (b) $y^2 – 64$ (c) $2x^2 + 5x – 3$ (d) $6t^2 – 13t + 5$

3. Simplify
   (a) $\frac{4x^3y}{8x^2y^3}$ (b) $\frac{3x+2}{3} + \frac{4x-1}{6}$

4. Solve the following equations
   (a) $\frac{h-1}{4} + \frac{3h}{5} = 4$ (b) $x^2 – 8x = 0$ (c) $p^2 + 4p = 12$

5. Simplify or evaluate the following:
   (a) $(x^2y)^3$ (b) $\frac{x^5}{x^{-2}}$ (c) $(\frac{8}{27})^{\frac{1}{3}}$

6. Solve the simultaneous equations $3x – 5y = -11$
   $5x – 2y = 7$

7. Rearrange the following equations to make x the subject
   (a) $v^2 = u^2 + 2ax$ (b) $V = \frac{1}{3}\pi x^2 h$ (c) $y = \frac{x+2}{x+1}$

8. Write the following in the simplest surd form:
   (a) $\sqrt{200}$ (b) $2\sqrt{2} + \frac{6}{\sqrt{2}}$ (c) $(\sqrt{3} + 5)(\sqrt{3} – 5)$

9. Solve $5x^2 – x – 1 = 0$ giving your solutions in surd form

10. Rationalise the denominator $\frac{\sqrt{3}+6}{5+\sqrt{3}}$

11. Prove that the sum of the squares of any three consecutive numbers is always one less than a multiple of 3.

Further Maths Practice Test – answers to be submitted in the 1st lesson

Your test in week 2 will ask similar questions to this test.

You may NOT use a calculator

If $ax^2 + bx + c = 0$ then $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

1. Expand and simplify
   (a) $(2a + 3)^2$ (b) $4x(3x - 2) - x(2x + 5)$ (c) $(y^3 + 4)(y^3 - 4)$

2. Factorise
   (a) $y^2 - 64$ (b) $2x^2 + 5x - 3$ (c) $6t^2 - 13t + 5$ (d) $6pq - 12p - 3q + 6$

3. Simplify
   (a) $\frac{4x^3y}{8x^2y^3}$ (b) $\frac{3x+2}{3} + \frac{4x-1}{6}$

4. Solve the following equations
   (a) $\frac{h-1}{4} + \frac{3h}{5} = 4$ (b) $p^2 + 4p = 12$ (c) $\frac{1}{x^2} + \frac{4}{x} + 3 = 0$

5. Simplify or evaluate the following:
   (a) $(x^2y)^3$ (b) $\left(\frac{8}{27}\right)^{\frac{1}{3}}$ (c) $\left(\frac{2x^{-5}}{x^{-3}}\right)^{-2}$

6. Solve the simultaneous equations $3x - 5y = -11$
   $5x - 2y = 7$

7. Rearrange the following equations to make x the subject
   (a) $v^2 = u^2 + 2ax$ (b) $y = \frac{x+2}{x+1}$ (c) $y = z - \frac{1}{\sqrt{x}}$

8. a) Find the value of $\frac{5\sqrt{6}}{\sqrt{27}} + \sqrt{8}$ in the form $r\sqrt{n}$, where r is a rational number and n is an integer.

   b)i) Simplify $(\sqrt{5} - 1)(\sqrt{5} + 1)$

   ii) Hence rationalise the denominator of $\frac{2\sqrt{5}+1}{\sqrt{5}+1}$

9. Solve $5x^2 - x - 1 = 0$ giving your solutions in surd form

10. Prove that the sum of the squares of any three consecutive numbers is always one less than a multiple of 3.

SOLUTIONS TO THE EXERCISES

CHAPTER 1:

Ex A

1) $28x + 35$ 2) $-15x + 21$ 3) $-7a + 4$ 4) $6y + 3y^2$ 5) $-4x - 4$
6) $7x - 1$ 7) $x^2 + 5x + 6$ 8) $t^2 - 7t + 10$ 9) $6x^2 + xy - 12y^2$
10) $4x^2 + 4x - 24$ 11) $4y^2 - 1$ 12) $12 + 17x - 5x^2$

Ex B

1) $x^2 - 2x + 1$ 2) $9x^2 + 30x + 25$ 3) $49x^2 - 28x + 4$ 4) $x^2 - 4$
5) $9x^2 - 1$ 6) $25y^2 - 9$

CHAPTER 2

Ex A

1) 7 2) 3 3) 1½ 4) 2 5) -3/5 6) -7/3

Ex B

1) 2.4 2) 5 3) 1 4) ½

Ex C

1) 7 2) 15 3) 24/7 4) 35/3 5) 3 6) 2 7) 9/5 8) 5

Ex D

1) 34, 36, 38 2) 9.875, 29.625 3) 24, 48

CHAPTER 3

1) $x = 1, y = 3$ 2) $x = -3, y = 1$ 3) $x = 0, y = -2$ 4) $x = 3, y = 1$
5) $a = 7, b = -2$ 6) $p = 11/3, q = 4/3$

CHAPTER 4

Ex A

1) $x(3 + y)$ 2) $2x(2x - y)$ 3) $pq(q - p)$ 4) $3q(p - 3q)$ 5) $2x^2(x - 3)$ 6) $4a^3b^2(2a^2 - 3b^2)$
7) $(y - 1)(5y + 3)$

Ex B

1) $(x - 3)(x + 2)$ 2) $(x + 8)(x - 2)$ 3) $(2x + 1)(x + 2)$ 4) $x(2x - 3)$ 5) $(3x - 1)(x + 2)$
6) $(2y + 3)(y + 7)$ 7) $(7y - 3)(y - 1)$ 8) $5(2x - 3)(x + 2)$ 9) $(2x + 5)(2x - 5)$ 10) $(x - 3)(x - y)$
11) $4(x - 2)(x - 1)$ 12) $(4m - 9n)(4m + 9n)$ 13) $y(2y - 3a)(2y + 3a)$ 14) $2(4x + 5)(x - 4)$

CHAPTER 5

Ex A

1) $x = \frac{y+1}{7}$ 2) $x = 4y - 5$ 3) $x = 3(4y + 2)$ 4) $x = \frac{9y + 20}{12}$

Ex B

1) $t = \frac{32rP}{w}$ 2) $t = \pm\sqrt{\frac{32rP}{w}}$ 3) $t = \pm\sqrt{\frac{3V}{\pi h}}$ 4) $t = \frac{P^2 g}{2}$ 5) $t = v - \frac{Pag}{w}$ 6) $t = \pm\sqrt{\frac{r-a}{b}}$

Ex C

1) $x = \frac{c-3}{a-b}$ 2) $x = \frac{3a + 2k}{k - 3}$ 3) $x = \frac{2y + 3}{5y - 2}$ 4) $x = \frac{ab}{b - a}$

CHAPTER 6

1) a) -1, -2 b) -1, 4 c) -5, 3 2) a) 0, -3 b) 0, 4 c) 2, -2
3) a) -1/2, 4/3 b) 0.5, 2.5 4) a) -5.30, -1.70 b) 1.07, -0.699 c) -1.20, 1.45
d) no solutions e) no solutions f) no solutions

CHAPTER 7

Ex A

1) $5b^6$ 2) $6c^7$ 3) b^3c^4 4) $-12n^8$ 5) $4n^5$ 6) d^2 7) a^6 8) $-d^{12}$

Ex B

1) 2 2) 3 3) 1/3 4) 1/25 5) 1 6) 1/7 7) 9 8) 9/4 9) ¼ 10) 0.2 11) 4/9 12) 64

13) $6a^3$ 14) x 15) xy^2

CHAPTER 8

Ex A

1) 9 2) 6 3) 2.5 4) 7.5 5) $3\sqrt{3}$ 6) $5\sqrt{2}$

7) 7 8) $9\sqrt{6}$ 9) 6 10) $5\sqrt{2} + 2\sqrt{5}$ 11) $14 + 12\sqrt{2}$

Ex B

1) $\frac{4\sqrt{3}}{3}$ 2) $\frac{\sqrt{10}}{10}$ 3) $\frac{8\sqrt{21}}{7}$ 4) $3\sqrt{5} - 1$

5) $7\sqrt{3}$ 6) $4\sqrt{5} + \sqrt{2}$ 7) $12\sqrt{5} + 4\sqrt{2}$ 8) $\frac{25-5\sqrt{7}}{8}$ 9) $\frac{4+\sqrt{2}}{7}$

10) $\frac{6+3\sqrt{2}+2\sqrt{7}+\sqrt{14}}{2}$ 11) $\frac{5-\sqrt{5}}{2}$

CHAPTER 9

1) (2n)+(2n+2)=4n+2, we cannot factorise 4, so the sum cannot be a multiple of 4.
2) (2n+1)+(2n+3)=4n+4=4(n+1), which is a multiple of 4.
3) The four consecutive numbers could be represented as n, n+1, n+2 and n+3
   Product of middle numbers $(n+1)(n+2)=n^2+3n+2$
   Product of first and last $n(n+3)=n^2+3n$
   Therefore, it is shown that the first product is two more than the second product.
4) Prove that the difference between two consecutive square numbers is always odd.
   $n^2+(n+1)^2=n^2+n^2+2n+1=2n^2+2n+1=2(n^2+n)+1$, which is an odd number.